TWO SIGMA

Street View

SEPTEMBER 2020

TWO SIGMA CLIENT SOLUTIONS TEAM

EXECUTIVE SUMMARY

Estimating forward-looking asset class returns is an unavoidable necessity for investors constructing portfolios. Reverse optimization is an estimation method that harnesses the wisdom of crowds by attempting to discern the forward-looking return views of all market participants from global portfolio holdings. In this Street View, we use reverse optimization to build forward-looking return estimates for major asset classes and compare their forecasted returns to their historical realized returns. Our key finding is that market participants appear to be expecting the next decade to look a lot different than the last.

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Inside:
Estimating Global Investor Views with Reverse Optimization

ESTIMATING GLOBAL INVESTOR VIEWS WITH REVERSE OPTIMIZATION

"It is difficult to make predictions, especially about the future."[1] Yet predictions are an unavoidable necessity for any investor seeking to position their portfolio for 2020 and beyond. Any portfolio weighting decision should account for expectations of risk and return.

This leaves us with two sets of predictions: risk (correlation and volatility) and return. Risk is relatively easy; return is harder. Risk tends to be strongly persistent through time, and long-term risk is predictable using even very simple models.[2] While we believe that portfolio construction should rely heavily on risk inputs and diversification, the need for return forecasts remains unavoidable.[3] So this still leaves us as investors with the considerably difficult task of estimating future returns.

Since any method of portfolio construction requires either explicit or implicit return assumptions, we prefer to make them explicit, so these assumptions can be examined and deemed sensible before driving critical investment decisions. What options do we as investors have at our disposal in determining our long-term return expectations? Differing asset definitions, data sets, and methodological details can result in a vast diversity of individual models and estimates, but we think most boil down to one of the following estimation archetypes:

1. **Estimating long-run return premia from historical data,** as done in our whitepaper Forecasting Factor Returns.[4]

2. **Building up forecasts from current valuations,** combining estimates for sub-components of longer-term returns such as income and growth expectations.

3. **Harnessing the wisdom of crowds,** by surveying or estimating the average forward-looking views of market participants.

This Street View delves into a particular quantitative example of the third method: harnessing the wisdom of crowds. By applying a concept called reverse optimization to the worldwide investable asset portfolio, we show how long-term factor and asset class returns can be estimated from the **aggregated allocation decisions** of all investors.

We believe this exercise can help investors in a couple of ways. First, the implied return estimates for key risk factors and asset classes derived from global market allocations can be used as an input when formulating your own capital market assumptions, a key input into asset allocation decisions. Second, while we apply this reverse optimization methodology to the global market portfolio, an investor could perform this analysis on their own portfolio to see what their own implied expectations are for future returns based on their existing asset allocation.

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1 An unfortunately unattributable witticism, most likely translated from the proceedings of the Danish Parliament, or Folketing (see https://quoteinvestigator.com/2013/10/20/no-predict/).
2 For a more concrete explanation of why we consider risk to be much more predictable than returns, please see our brief digression in Appendix 1.
3 Even risk-based approaches to portfolio construction such as minimum volatility or risk parity require return assumptions to drive the portfolio weights. Minimum volatility, in particular, is equivalent to mean-variance optimization where one assumes all assets have equal expected return, while risk parity implicitly assumes a more complex relationship where higher risk assets (as measured by both variance and their covariance with other assets) have proportionally higher expected return. Finally, at a minimum, we believe investors should understand the risks in their portfolio and whether there are expected long-term positive return premia associated with those risks. This is a key step because it is not true that all risks come hand in hand with positive excess returns, as mentioned in "Risk Without Return" (Nigro, 2019).
4 "Forecasting Factor Returns" (Duncombe, Nigro, and Kay, 2019)

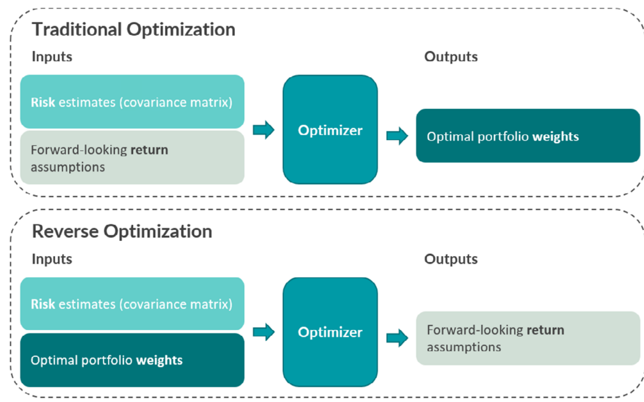
We'll begin the Street View by setting up the reverse optimization problem and defining one of its key inputs: the **global investable market portfolio**. In the next section, we will uncover the outputs from the reverse optimization and translate them into a more usable format by establishing a bottoms-up "anchor" return estimate for global equity. Finally, we will estimate the long-term implied returns for several asset classes and see how they compare to realized returns over the past decade.

Setting Up a Reverse Optimization of the Global Market Portfolio

In order to harness the wisdom of crowds, we need to collect the forward-looking views of market participants. One way to collect this information would be through a survey. Unfortunately, surveys come with a variety of issues, including logistical hurdles and a host of biases (e.g., selection and response bias[5]). In contrast, the reverse optimization approach allows us to harness the aggregate market's wisdom directly from the bets that have real money behind them.

Reverse optimization begins with the assumption that investors are each individually attempting to allocate their portfolios in a fairly "optimal" fashion, aiming to meet their objectives based on individual risk and return assumptions. These collective actions aggregate to push asset prices around until their relative levels approximately result in an optimal allocation based on the marginal investor's forward-looking views.[6] Thus we can think of the global market portfolio, with market capitalization of each asset serving as the "weights," as the aggregate of investors' best guess at the optimal portfolio. Multiplying those "optimal weights" by the covariance matrix (i.e., risk estimates) of the assets will then generate the forward-looking returns for each factor that would result in the market portfolio being optimal.[7] Hence the name "reverse optimization," as this procedure reverses the typical process of combining risk estimates with return estimates to generate optimal weights.

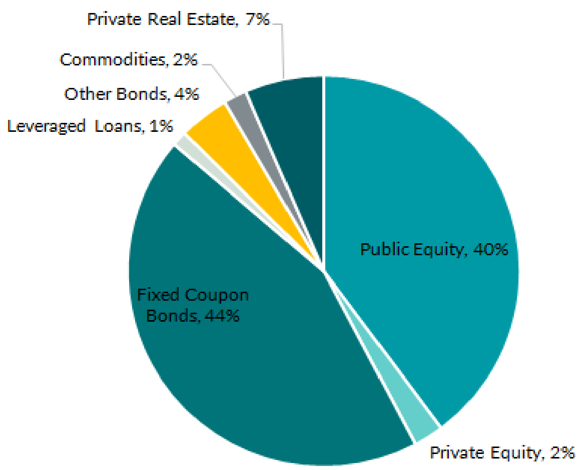
Exhibit 1: Illustration of the Differences Between Traditional and Reverse Optimization



To estimate the historical risk and current allocations of the global market portfolio, we draw upon the work of Doeswijk, Lam, and Swinkels (2019), who identified key investable asset categories to represent basically "all assets held by financial investors around the globe."[8] Overall, our proxy for the global portfolio contains twelve categories covering public and private equity, an array of fixed-income instruments, commodities, and institutional real estate holdings.[9] Exhibit 2 displays the high-level portfolio weights, and Appendix 2 has more construction details.

5 Selection bias is the bias that is introduced when the sample surveyed is not fully representative of the entire population. This bias can occur if there is voluntary participation, in which case our sample might not be reflective of all market participants. Response bias is the bias that is introduced if survey participants do not answer honestly or accurately. Survey participants could try to throw the answers off to preserve their "edge" in return prediction.
6 Our reverse optimization exercise is essentially assuming that markets are efficient. While investors may have different objective functions, we are assuming that the marginal investor's objective is to maximize risk-adjusted returns (or Sharpe Ratio). To the extent that a meaningful proportion of market participants have an objective that is not maximizing Sharpe Ratio, this reverse optimization exercise can still be useful as a way for a Sharpe Ratio-maximizing investor to measure which assets might be mispriced.
7 In particular, this math works to generate return forecasts if we assume that a portfolio is "mean-variance optimal", which is a common framework because 1) it provides a reasonable local approximation of most investors' actual risk aversion (though strictly speaking, it assumes either Gaussian return distributions or what is known as quadratic utility), 2) the problem formulation should not have any constraints to satisfy, and 3) the math is relatively straightforward. We do not presume to say how much of mean-variance optimization's prevalence is respectively due to each of these considerations.
8 Doeswijk, R., T. Lam, and L. Swinkels (2019). "Historical Returns of the Market Portfolio", Review of Asset Pricing Studies, forthcoming. Accessible at https://doi.org/10.1093/rapstu/raz010
9 Nearly all of these asset classes can be well approximated with indices based on current market prices. For estimating the risk of Private Equity, we used a simplifying assumption of 1.4 beta to the global public equity market based on results from Ang et al. (2018). For Private Real Estate, we used de-smoothed quarterly returns to the Preqin Real Estate Index.

Exhibit 2: Estimated Global Market Portfolio Weights
See Appendix 2 for construction details and sources.



Once we have our estimated historical returns for the global market portfolio, we can decompose the market portfolio's exposures to factors in order to generate factor-level return forecasts rather than asset class forecasts. Forecasting factor expectations rather than asset classes allows a more flexible application of the reverse optimization results, as it provides return forecasts at a more elemental and orthogonal level. If we want to generate a forecast for a sub-class of assets such as U.S. high yield bonds, for example, we can add up the return forecasts for the macro factors driving that sub-class rather than requiring that the sub-class be split out as its own category within the global market portfolio.

This is how we can transform the reverse optimization results for a global portfolio based on extremely broad, comprehensive indices into a diverse array of individual asset class forecasts.

We decided to use the Two Sigma Factor Lens™ [10] as our factor risk model, which was constructed with the aim of accurately and intuitively describing risk drivers in investor portfolios. The factor lens was designed to be holistic, in that it intends to capture the large majority of cross-sectional and time-series risk for typical portfolios. This characteristic holds true for the global market portfolio, as seen in the resulting factor decomposition in Exhibit 3. Our use of factor loadings instead of the individual asset classes does not appear to lose any significant information, with over 97% of the variance in the global market portfolio explained by its estimated factor loadings.[11]

Exploring the results in Exhibit 3 in more detail, the Equity factor was unsurprisingly dominant, driving 94% of the portfolio's risk.[12] The Interest Rates factor beta was meaningful, but the contribution to risk was negative, implying this exposure was a diversifier for the overall portfolio (see more on this in footnote 14). The Credit factor's small presence may be initially surprising given the large capitalization of global credit bonds, but this result can be explained by 1) the relatively lower volatility of bonds versus equities, and 2) the fact that it's a residual Credit factor intended to isolate the unique risk of the asset class by extracting the level of Equity and Interest Rates risk embedded in corporate bonds. In other words, the factor hedges out sensitivities of corporate bonds to fundamental drivers of risk such as changes in risk-free discount rates and aggregate investor risk aversion, resulting in a purified Credit risk factor.[13]

Exhibit 3: Global Market Portfolio Factor Exposures and Annualized Factor Risk Contributions
See Appendix 2 for construction details and sources.

Factor	Factor Exposure (beta)	Factor Risk Contribution
Equity	0.49	94.1%
Interest Rates	0.30	-1.0[14]
Credit	0.06	4.2%
Commodities	0.03	0.3%
Residual	-	2.4%

10 This analysis focused on the macro factors that are outlined in the whitepaper "Introducing the Two Sigma Factor Lens" (Duncombe and Kay, 2018). Please note that the Credit and Commodities factors are residualized to the Equity and Interest Rates factors. We excluded several factors, including those that 1) were not significant in the liquid portfolio (i.e., ex Private Real Estate), 2) are "local" and therefore specific to investors in a particular country or region, and 3) were recategorized to a style category (Equity Short Volatility was recategorized since the publication of the whitepaper "Introducing the Two Sigma Factor Lens" [Duncombe and Kay, 2018] from a secondary macro factor to a macro style factor). While research on the Two Sigma Factor Lens™ plays a foundational role in the development of the Venn® Platform, any use by Venn of the Two Sigma Factor Lens™ can differ materially from the content, research, or methodologies discussed herein.
11 Nearly all the residual risk can be explained by the Private Real Estate component, as our macro factors could only explain 31% of the variance in that series through time. However, this asset class is a small component of the global market portfolio's risk, contributing less than 10% to the global portfolio's volatility historically.
12 Despite a 44% capital allocation to bonds, Equity risk still overshadowed that of Interest Rates, as equity volatility was more than four times higher than bond volatility over this period.
13 Read the whitepaper "Introducing the Two Sigma Factor Lens" (Duncombe and Kay, 2018) for more information on the Credit factor's construction.
14 The Interest Rates factor's correlation-adjusted contribution to risk was negative. This result implies that the factor's contribution to the global market portfolio's return was negatively correlated with the global market portfolio's overall return.

One Final Step: Anchoring Returns

Crucially, the implied returns that come from reverse optimization are only relative return levels - not absolute return expectations. That is, reverse optimization may tell us that investors expect twice the return from factor A as from factor B, but provides no guidance on whether those expected returns are 2% versus 1% annually or 20% versus 10%.

To estimate the absolute level of expected returns for each factor, we need either to know investors' true expectations for the portfolio as a whole or the expectation for any single factor from which to calibrate the remaining factors. Since we have a fundamental understanding of public equity returns from the dividend discount model (discussed in more detail below), we will use it as our base factor for the other factors.

Once we establish a forecast range for the returns to the Equity factor, we can use upper, lower, and central estimates to rescale the relative factor returns calculated from the global portfolio's weights in our previous section. This rescaling provides the matching ranges of annualized return forecasts for the Interest Rates, Credit, and Commodities factors previewed in Exhibit 4.

Exhibit 4: Annual Real Return Forecasts[15] from Rescaled Reverse Optimization[16]

	Equity	Interest Rates	Credit	Commodities
Top of Range	6.6%	-0.0%	2.5%	0.3%
Central Est.	5.3%	-0.1%	2.0%	0.3%
Bottom of Range	2.8%	-0.1%	1.1%	0.1%

How did we arrive at the base return estimates for Equity that are then used to determine the return estimates for the other factors? As mentioned earlier, we relied on the dividend discount model,[17] which suggests that equity returns come from three sources:

1. Cash flows paid to shareholders (i.e., dividends and net buybacks)
2. Growth in cash flows over time
3. Changes in the price multiple of shares relative to their underlying cash flows

The range of potential forward-looking equity views presented in the first column of Exhibit 4 accounts for both conservative and aggressive assumptions about sustainable payouts (#1) and growth rates (#2). In regards to #3, we assume no change in valuations.[18] Let's discuss the data and assumptions for both #1 and #2 and how these come together to arrive at a forecast return for Equity.

#1: Estimating Cash Flows Paid to Shareholders

To estimate a range of sustainable cash flows to investors, we opt to start from earnings in order to abstract away from companies' decisions between share buybacks versus dividends as the means of cash return. Our selected measure of earnings is the 10-year trailing average of inflation-adjusted earnings per share on the MSCI All-Country World Index (in USD), to provide global coverage and smooth business cycle variations.[19] As of June 2020, this produced a cyclically-adjusted global equity earnings yield of 5.3%. This suffices as a simple point estimate for forward-looking equity real returns and will serve as our central return estimate for Equity, as shown in Exhibit 4.[20]

15 It's important to acknowledge that the return forecasts in Exhibit 4 are real (i.e., adjusted for inflation) and gross of cash. Whereas, reverse optimization actually predicts the relative excess returns over cash. This means that a forecast for the real cash rate would need to be subtracted from the return estimates in Exhibit 4 to arrive at excess returns. For reference, real cash rates (as measured by the annual yields on U.S. TIPS from five to ten years' maturity) have moved around quite a bit over the last six months or so, especially given the market activity surrounding the COVID-19 pandemic. In late 2019, they were around zero, so we did not have to worry much about the distinction between gross and excess of cash returns. As of July 2020, they were ~-1%.

16 Again, it's important to note that the Credit and Commodities factors are residualized to the Equity and Interest Rates factors.

17 The dividend discount model equation is as follows: the rate of global equity return is equal to 1) the value of dividends (or aggregate stock payouts) divided by the current price (or current market cap of all stocks) plus 2) the expected growth rate of dividends (or aggregate stock payouts). Note that 1) reflects the price multiple of shares using the dividend yield (or aggregate stock payout yield). For more on the dividend discount model, see Gordon, M.J and Eli Shapiro (1956) "Capital Equipment Analysis: The Required Rate of Profit," Management Science, 3,(1) (October 1956) 102-110.

18 We are in accord with research from others in the industry that assume no mean reversion. For example, see the discussion on Historical Perspective and Mean Reversion in AQR's 2015 Capital Market Assumptions for Major Asset Classes. To summarize, their reasons behind assuming no mean reversion include statistical evidence showing a noisy relationship for multi-year mean reversion in the real yields of U.S. equities as well as potential structural changes that may have an impact on the go-forward relationship between historical and future real yields.

19 The average of historical inflation-adjusted earnings is multiplied by 1.075 to get our final inflation-adjusted measure in order to reflect some degree of expected real growth in earnings over the 10 year period.

20 This estimate of 5.3% real returns for global equities carries a couple of (very strong) assumptions, including: 1) public companies' equity capital in aggregate generates a constant earnings yield that grows with inflation, and 2) earnings not paid out to shareholders are reinvested with an expected real return equal to the cost of capital (i.e., new business investment has an overall rate of return matching the equity market's expected return).
The estimate is also very close to the estimated excess returns produced by the historical risk premium model outlined in the whitepaper "Forecasting Factor Returns" (Duncombe, Nigro, and Kay, 2019).

In order to generate the top and bottom range estimates, we can use historical data to determine approximately what percentage of aggregate earnings represents free cash flows that can be paid to investors (versus those earnings that have gone toward funding future growth through capital expenditures or working capital increases). Data in Appendix 3 shows that the level of payouts has averaged somewhere between 40-70% of corporate earnings. Given our cyclically-adjusted earnings yield of 5.3% on global equities, this translates into a range of sustained payouts to shareholders between 2.1% to 3.7% of current equity prices.

#2: Estimating Growth Rates in Cash Flows

In order to generate forward-looking return assumptions for global equities (and the other factors), we cannot stop at current payout yields. We also need to take into account the expected growth in payouts over time. Here we will again keep things simple and establish a conservatively broad range for potential payout growth rates.

The lower end of our forecast range for long-term real payout growth will come from Bernstein and Arnott's "Earnings Growth: The Two Percent Dilution" (2003), which documented that dividends per share growth for major equity markets substantially trailed their respective national GDP growth over the twentieth century.[21] Their paper estimated a long-term average real dividend per share growth of a mere 0.7% annually.[22]

The higher end of our forecast range for long-term real payout growth will come from Straehl and Ibbotson (2017), who also analyzed the average level and real growth rates for dividends on U.S. equities for the period 1871-2014, and came to a very different conclusion.[23] Their paper found that the historical growth rate to be applied to dividend yield should sit somewhere between real GDP growth and real GDP growth per capita.[24] So, for the high end of our global payout growth range, we take the International Monetary Fund's (IMF's) forecast for long-term global real GDP growth (at market exchange rates) of 2.9%.[25]

Combining #1 and #2 to Arrive at a Forecast Range for Equity

Now that we have our aggressive and conservative assumptions around payout ratios (#1) and growth rates (#2), we can form a range of real equity returns around the central, earnings-based estimate of 5.3%. Exhibit 5 displays the build-up of these numbers into our final return predictions for the global Equity factor.

Exhibit 5: Earnings-Based Real Return Forecast Build-Up for Global Equity, Starting with a 5.3% Cyclically-Adjusted Earnings Yield

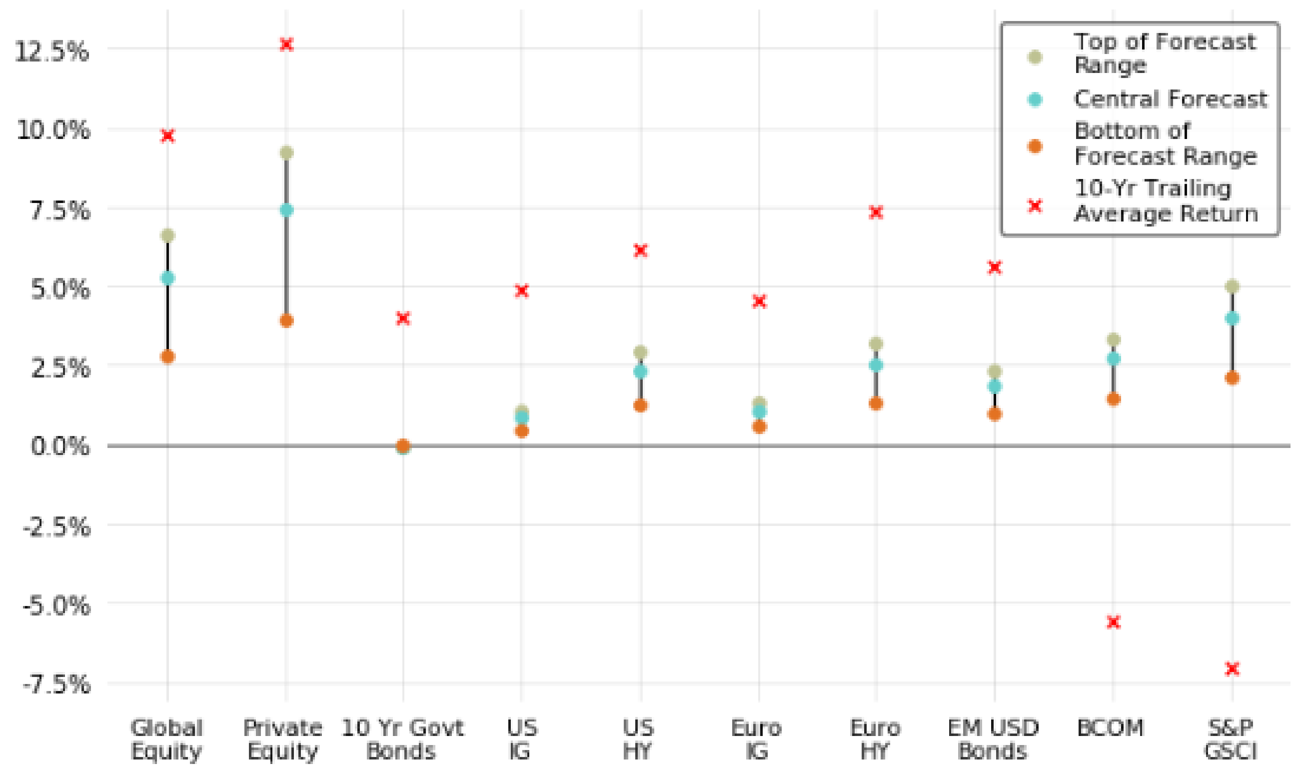
	(#1) Payout Yield (Real Payout Ratio * Cyclically-Adjusted Earnings Yield)	(#2) Real Payout Growth Rate	Estimated Real Return (#1 + #2)
Top of Range	(70% * 5.3%) = 3.7%	2.9%	6.6%
Central Est.	—	—	5.3%
Bottom of Range	(40% * 5.3%) = 2.1%	0.7%	2.8%

From Factor Forecasts to Asset Forecasts

With the factor forecasts implied by our reverse optimization, we can then build up individual forecasts for key asset classes (as well as any individual asset) based on their exposures to the underlying macro factors.[26] Using the factors in the Two Sigma Factor Lens™, we calculate the factor betas for each asset class in our portfolio using returns from the past ten years (July 2010 - June 2020), then multiply each asset's factor betas by the factor return forecasts from Exhibit 4 to calculate the asset class's return forecasts. The results are shown in Exhibit 6.

21 Bernstein, W.J. and R.D. Arnott (2003). "Earnings Growth: The Two Percent Dilution", Financial Analysts Journal, 59(5), 47-55.
22 See Table 1 of Bernstein, W.J. and R.D. Arnott (2003). "Earnings Growth: The Two Percent Dilution", Financial Analysts Journal, 59(5), 47-55.
23 Straehl, P.U. and R.G. Ibbotson (2017). "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy", Financial Analysts Journal, 73(3), 32-52. The crux of their argument is that a total return investor who reinvests dividends will effectively end up holding more shares through time, and thus the proper dividend growth rate for estimating total returns to equities should include this growth on reinvested income.
24 Straehl, P.U. and R.G. Ibbotson (2017). "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy", Financial Analysts Journal, 73(3), 32-52. More precisely, they found the growth rate that should be applied to "Total Yield," incorporating both dividends and share buybacks but excluding new share issuance, is in line with real GDP growth per capita. The growth rate for "Net Total Yield," which also accounts for dilution from new share issuance, is in line with overall real GDP growth. The growth rate applicable solely to dividend income sat roughly in between these two.
25 The 2024 projection for "World Growth Rate Based on Market Exchange Rates" from Table A1 in the IMF's October 2019 World Economic Outlook. Due to the high level of uncertainty in global economic conditions, the April 2020 World Economic Outlook did not include projections beyond 2021.
26 Asset class forecasts assume that all return premium comes from the asset class' factor exposures and none from its residual risk.

Exhibit 6: Expected Long-Term Real Returns by Asset Class[27]
Time period: July 2010 - June 2020 (for 10-Yr Trailing Average Return and the calculation of each asset class' factor betas).



A Tougher Decade Ahead: Findings Summary and Conclusion

Despite our best efforts to show a wide range of possible return expectations, even the most optimistic end of the range pales compared to the fortunate 2010s. The decade to come appears unlikely to match the nearly universal rise of risky assets over the past ten years, though those still keeping faith in the diversification potential of commodity futures may have cause to rejoice! The 2020s are certainly off to an unexpected start with the precipitous fall and (partial) recovery (as of the date of this writing) of many risk assets, including equities, credit, and oil, due to the COVID-19 pandemic.

Here are a few notable observations from our findings:

- The results suggest that future returns will be lower than the average for the past 10 years for almost all asset classes examined. This may indicate that investors are not expecting a future in-line with past performance when evaluating go-forward returns.
- A similarity with the past is that investors continue to expect a large premium of equities over 10-year bonds (in this case, greater than a 5% annual premium using the central estimates). This result implies that the equity risk premium puzzle[28] (i.e., the historical outperformance of stocks over 10-year bonds is above and beyond that which would be expected to compensate rational, risk-averse investors) still appears to be relevant on a go-forward basis.
- Investors appear to expect to lose money (in real terms) from holding bonds, as evidenced by the negative premium for the Interest Rates factor, which is represented by 7-10 year

27 The trailing 10-year return for Private Equity uses the 10-year return through Q4 2019 for the Preqin Private Equity Index, minus the contemporaneous annualized return to a U.S. dollar cash index.
28 Mehra, Rajnish and Edward C. Prescott (1985). "The Equity Premium: A Puzzle", Journal of Monetary Economics, 15, 145-161.

global government bonds.[29] Despite a low starting point, it's certainly possible for real yields to drop further and even go sharply negative. This result is also in line with econometric models that seek to predict the term premium embedded in longer-term bonds from the shape of the yield curve.[30]

To sum up, reverse optimization is a useful method that can help "harness the wisdom of crowds." It aims to derive the implicit return assumptions of market participants in aggregate by using the total market caps of global investable assets, exploiting the high predictability of risk, and relying on the plain math of mean-variance optimization.

Finally, while this Street View applies the reverse optimization method to the global market portfolio in order to estimate aggregated global investor outlooks for different asset classes, another application of the technique is to apply it to individual portfolios to backout implied views on factors or individual holdings. Comparing these implied views to other return estimation methods described at the beginning of the Street View, such as long-term realized returns or investors' capital market assumptions, could identify areas of misalignment between actual portfolio positioning and forward-looking return expectations.

29 Despite the negative expected return, current bond holdings are substantial (as displayed in Exhibit 2), and their share of the global market portfolio could increase further if net bond issuance ticks up due to low financing costs (i.e., low yields). Individual investors will still be motivated to hold low-yielding bonds if they have an off-consensus return view (remember, this result is an aggregated investor outlook) or if they have non-return seeking objectives, such as liability hedging, lower volatility, and/or diversification. Note that any increase in the weight of bonds in the global market portfolio wouldn't necessarily correspond to higher expected returns for the asset class, as volatility and correlations are also taken into account in the reverse optimization problem.

30 "Three Factor Nominal Term Structure Model" (Board of Governors of the Federal Reserve System, 2019) updated on November 5, 2019.

APPENDIX & DATA SOURCES

Appendix 1: A Brief Digression on Predicting Risk Versus Predicting Return

We simply asserted in the introduction that risk is easy to predict, but we want to show our work here by testing a simple model to predict asset volatility:

1. We began with the daily return histories for 300+ instruments in Two Sigma's broader macro universe, which includes rolled returns to futures and forwards contracts (as well as some cash securities) across global fixed income, equity index, commodity, and currency markets.

2. At each month-end where a security had at least three months (63 days) of trailing returns, we calculate the trailing standard deviation of returns using exponential weights with a three-month half-life.

3. At each month-end, we also calculate the equal-weighted standard deviation of returns for the following month (the future volatility being predicted).

4. We discard any observations with insufficient data (i.e., either historical or forward-looking volatility is missing) or with an annualized return standard deviation below 0.1% (as those likely represent observations without sufficient price discovery).

With over 60,000 observations, this single measure of per-instrument historical volatility manages to predict the next month's volatility with an r-squared of 0.83 (or correlation of 0.91)! For context, Two Sigma researchers spend their years (and considerable computing power) combining information from hundreds of datasets to try and predict returns. The combined returns prediction accuracy of all this human and machine effort is *much* smaller than that of our ten-second risk model.

Appendix 2: Construction Details of the Global Market Portfolio

Global Market Portfolio Weights

Asset Category	Estimated Market Cap ($T)	Weight	Market Cap Source
Public Equity	58.79	39.8%	Bloomberg (ticker M1WDIM, using free float market cap) as of 7/2020
Private Equity	3.65	2.5%	McKinsey Global Private Markets Review 2020 (excl. Private Real Estate) with 1/3 haircut to reflect "dry powder" capital not yet invested
Fixed Coupon Bonds	64.99	44.0%	Bloomberg (ticker LF93TRUU, using market cap) as of 7/2020
Inflation-Linked Bonds	3.46	2.3%	Bloomberg (ticker BCIW1A, using market cap) as of 7/2020
Leveraged Loans	1.80	1.2%	Bank of England (estimated amount held by non-bank institutional investors)
U.S. Floating Rate Bonds	0.38	0.3%	Bloomberg (ticker BFRNTRUU, using market cap) as of 7/2020
European Floating Rate Bonds	0.18	0.1%	Bloomberg (ticker LEF1TRUH, using market cap) as of 7/2020
Convertible Bonds	0.45	0.3%	Bloomberg (ticker BGCVTRUU, using market cap) as of 7/2020
U.S. Municipal Bonds	1.65	1.1%	Bloomberg (ticker LMBITR, using market cap) as of 7/2020
Gold	2.49	1.7%	World Gold Council using the value of Private Investment (i.e., bars & coins and ETFs)
Commodity Futures	0.32	0.2%	Barclays Commodity Investor
Private Real Estate	9.60	6.5%	MSCI Real Estate Market Size 2019

The Global Market Portfolio is intended to measure the worldwide investable portfolio. Therefore, in the case of the two commodity-related asset categories, Gold and Commodity Futures, note that we decided to focus on the value of gold and other commodities that are used for investment only rather than the value of all gold and commodities, which would include those that are used for actual consumption and/or production.

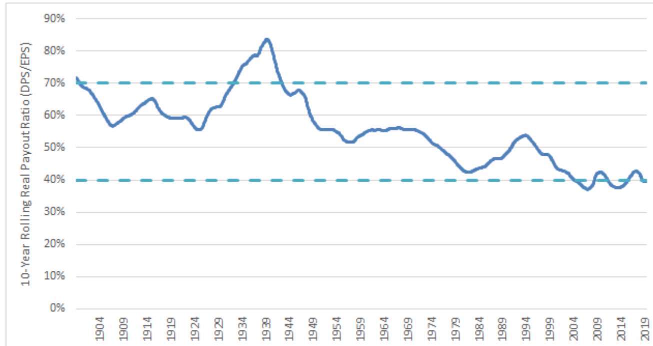
Global Market Portfolio Proxy Indexes

Asset Category	Returns Proxy Index	Bloomberg Ticker (if applicable)
Public Equity	MSCI ACWI IMI Net Total Return Local Index	MIMLAWON
Private Equity	Public Equity * 1.4	–
Fixed Coupon Bonds	Bloomberg Barclays Multiverse Total Return Index Value Hedged USD	LF93TRUH
Inflation-Linked Bonds	Bloomberg Barclays World Govt Inflation-Linked All Maturities TR Hedged USD	BCIW1U
Leveraged Loans	S&P Global Leveraged Loan Index	SPLGAL
U.S. Floating Rate Bonds	Bloomberg Barclays US Floating Rate Notes TR Index Value Unhedged USD	BFRNTRUU
European Floating Rate Bonds	Bloomberg Barclays EURO Floating Rate Notes TR Index Value Unhedged EUR	LEF1TREU
Convertible Bonds	Bloomberg Barclays Global Convertibles Composite Total Return Hedged USD	BGCVTRUH
U.S. Municipal Bonds	Bloomberg Barclays Municipal Bond Index Total Return Index Value Unhedged USD	LMBITR
Gold	Gold Spot Exchange Rate	XAUUSD
Commodity Futures	Bloomberg Commodity Index Total Return	BCOMTR
Private Real Estate	Preqin Real Estate Index	–

Appendix 3: Establishing the Historical Level of Payouts

The following exhibit shows the historical level of dividend payouts to investors as a fraction of corporate earnings using Robert Shiller's extension of the S&P 500 Index back to the late 19th century. Aside from a brief period around the Great Depression and most recently in the post-2000 era, the level of dividend payouts has averaged somewhere between 40-70% of corporate earnings. The increasing use of share buybacks to return capital (rather than dividends) helps explain the declining values of this series post-2000, which suggests that the total payouts including both buybacks and dividends remain within their historical range.

Rolling 10-Year Payout Ratio for S&P 500 Index[31]
Time period: January 1900 - March 2020.



Appendix 4: Proxy Indexes for Asset Classes in Exhibit 6

Asset Category	Returns Proxy Index	Bloomberg Ticker (if applicable)
Global Equity	Two Sigma Equity Factor * 1.0	–
Private Equity	Two Sigma Equity Factor * 1.4 (the simple assumption of 1.4 beta to the global public equity market is based on results from Ang et al. [2018])	–
10 Year Govt Bonds	Two Sigma Interest Rates Factor * 1.0	–
US IG	Bloomberg Barclays US Corporate Total Return Value Unhedged USD	LUACTRUU
US HY	Bloomberg Barclays US Corporate High Yield Bond Index	LF98TRUU
Euro IG	Bloomberg Barclays Pan European Aggregate Corporate TR Index Hedged USD	LP05TRUH
Euro HY	Bloomberg Barclays Pan-European High Yield Total Return Index Value Hedged USD	LP01TRUH
EM USD Bonds	J.P. Morgan EMBI Global Diversified Composite	JPEIDIVR
BCOM	Bloomberg Commodity Index Total Return	BCOMTR
S&P GSCI	S&P GSCI Total Return CME	SPGSCITR

31 Data from Robert Shiller's website, taking 10-year rolling averages of real dividends per share divided by real earnings per share.

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